Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 2

11 November 2025

Zone A morning | **Zone B** morning | **Zone C** morning

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

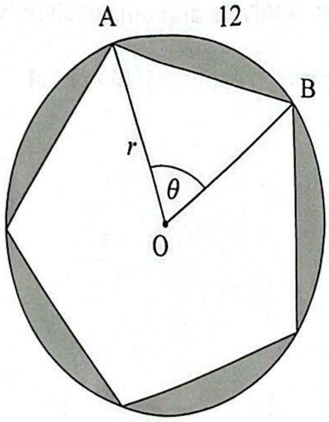
1. [Maximum mark: 6]

The following diagram shows a regular pentagon inscribed in a circle with centre O and radius r cm.

The angle $\hat{AOB}$ is θ, where θ is measured in radians.

The arc AB is 12 cm.

diagram not to scale



(a) Find

(i) θ;

(ii) r. [3]

(b) Find the area of the shaded region. [3]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 4]

A particular service group at a school has ten members.

The group intends to select a committee of seven by first choosing a chairperson, a vice-chairperson and a treasurer, then selecting four additional members to complete the committee.

Determine the number of ways in which this committee can be chosen.

3. [Maximum mark: 7]

Data was collected on two variables, x and y, from eight members of a population. The data is shown in the table below.

x	3.8	6.3	2.5	2.4	8.9	10.7	6.7	10.3
y	11.8	9.7	6.1	4.1	p	23.3	13.7	24.5

(a) Find $\bar{x}$. [2]

The regression line of y on x for this data is $y = 0.014375 + 2.1625x$.

(b) Find

(i) $\bar{y}$;

(ii) the value of p. [4]

(c) Hence, find Pearson's product-moment correlation coefficient, r. [1]

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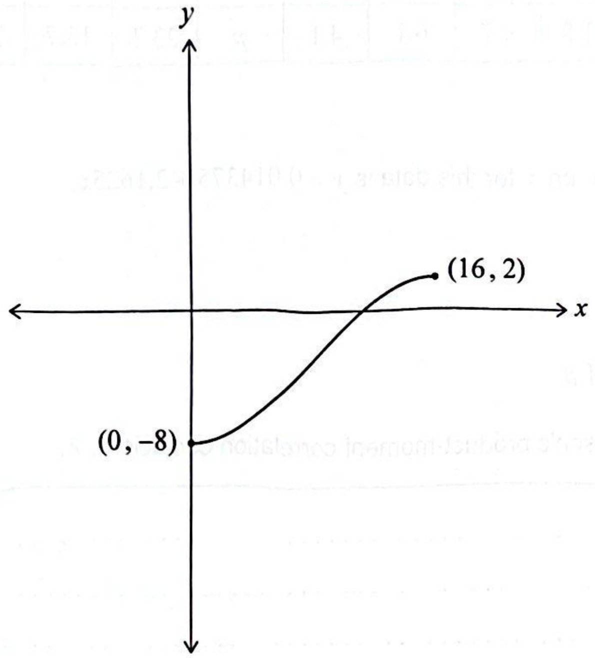
4. [Maximum mark: 7]

Consider the function $g(x) = a\cos(bx) - 3$, where $x \in \mathbb{R}$ and $a, b \in \mathbb{R}$.

The following diagram shows part of the graph of g.



The graph of g has a local minimum at $(0, -8)$ and a local maximum at $(16, 2)$.

(a) Find the value of

(i) a;

(ii) b, where $b > 0$. **[3]**

(b) Write down the smallest positive value of the constant k such that $f(x + k) = f(x)$ for all x. **[1]**

(c) The function $g(x)$ can be written in the form $f(x) = p\sin b(x - q) - 3$ where $p, q \in \mathbb{Z}^+$.

(i) Find the smallest positive value of q.

(ii) For this value of q, write down the value of p. **[3]**

(This question continues on the following page)

(Question 4 continued)

5. [Maximum mark: 5]

Consider the following grouped data set.

Class	**Frequency**
$15 < y \leq 20$	31
$20 < y \leq 25$	42
$25 < y \leq 30$	61
$30 < y \leq 35$	46
$35 < y \leq 40$	29

(a) Find an estimate for

(i) the mean;

(ii) the standard deviation. **[2]**

The data can be approximated by a normal distribution.

(b) Use this information and your mean and standard deviation found in part (a) to find an approximate value for the interquartile range correct to two significant figures. **[3]**

6. [Maximum mark: 5]

Consider the infinite series

$$\sum_{k=0}^{\infty}\left(\frac{53}{1000}\right)\left(\frac{1}{100}\right)^{k}.$$

(a) Find the exact value of S_{∞}. [3]

Let $0.2\overline{53}$ represent the repeating decimal $0.2535353\ldots$.

(b) Use your answer from part (a) to express $0.2\overline{53}$ as a fraction such that the numerator and denominator have no common factors. [2]

Please **do not** write on this page.

Answers written on this page will not be marked.

7. [Maximum mark: 7]

Two machines are used in a factory to manufacture semiconductors. Machine A manufactures defective semiconductors 10% of the time, while machine B manufactures defective semiconductors 5% of the time.

A randomly selected machine manufactures ten semiconductors.

Both machines are equally likely to be selected.

(a) Find the probability that exactly two semiconductors are defective. [4]

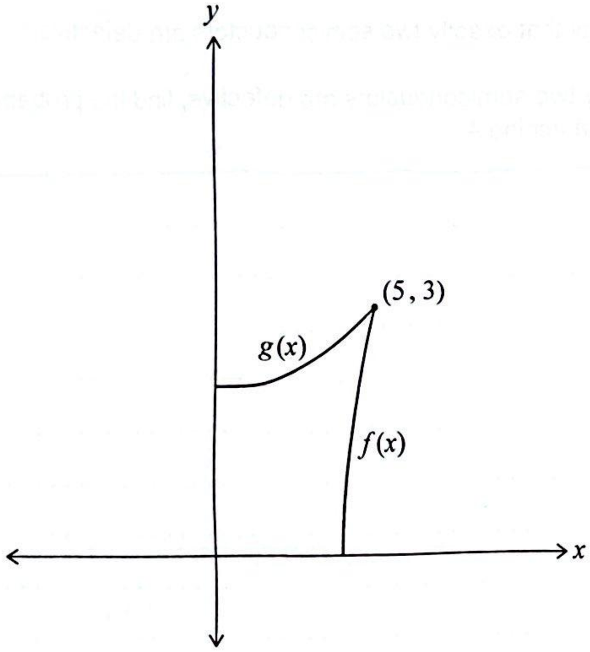
(b) Given that exactly two semiconductors are defective, find the probability that they were manufactured by machine A. [3]

8. [Maximum mark: 6]

The following diagram shows the graphs of two functions, f and g, where

$$f(x)=\sqrt{x^2-16} \text{ for } 4 \le x \le 5 \text{, and}$$

$$g(x)=2+\frac{x^2}{25} \text{ for } 0 \le x \le 5 .$$



The graphs of f and g intersect at the point $(5, 3)$.

The region bounded by the graphs of f, g and both axes is rotated $360°$ about the y-axis to form a solid of revolution.

Find the volume of the solid formed.

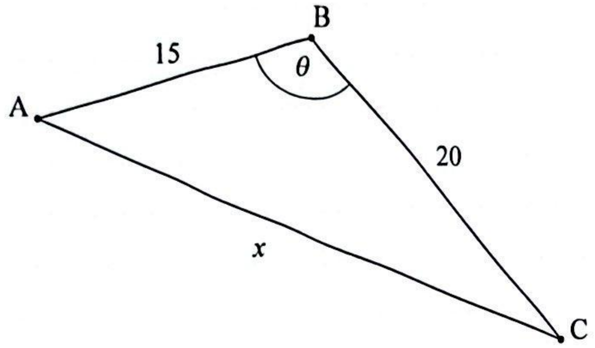
(This question continues on the following page)

(Question 8 continued)

9. [Maximum mark: 7]

Consider triangle ABC with $AB = 15\,\text{cm}$, $BC = 20\,\text{cm}$, $AC = x\,\text{cm}$ and $A\hat{B}C = \theta$ where $0 < \theta < \frac{\pi}{2}$.

diagram not to scale



Angle θ is decreasing at the constant rate of $\frac{\pi}{60}$ radians per minute.

Determine how fast the length of AC is changing when the area of the triangle is $140\,\text{cm}^2$.

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Do **not** write solutions on this page.

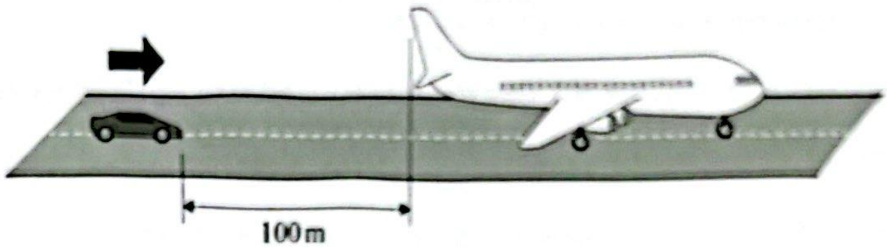
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 17]

An airplane lands on a runway 100 metres in front of a stationary car. At the instant the airplane lands, the car begins to travel in the same direction towards the airplane.

diagram not to scale



Let t represent the number of seconds after the airplane lands. For $t \geq 0$, the velocities of the airplane and the car in m s^{-1}, can be modelled by the following equations:

$$v_{air} = 60e^{-0.1t}$$

$$v_{car} = 5t$$

(a) When the airplane lands, write down the speed of

(i) the airplane;

(ii) the car. [2]

(b) Find

(i) the value of t when the airplane and the car have the same speed;

(ii) the speed at this time. [3]

Let $d(t)$ represent the distance, in metres, between the car and the back of the airplane after t seconds.

(c) If $d(0) = 100$, find $d(t)$. [7]

(d) Hence, find how long it takes for the car to reach the back of the airplane. [2]

(e) Find the distance travelled by the car when it reaches the back of the airplane. [3]

Do **not** write solutions on this page.

11. [Maximum mark: 18]

(a) Use integration by parts to find $\int \arccos x \, dx$. [4]

The probability density function of a random variable X, is given by

$$f(x) = \begin{cases} 3x\arccos\left(x^2\right), & 0 \le x \le k; \\ 0, & \text{otherwise} \end{cases}.$$

(b) (i) Show that $k^2 \arccos(k^2) - \sqrt{1-k^4} + \frac{1}{3} = 0$.

(ii) Hence, find the value of k. Give your answer correct to six significant figures. [6]

(c) Find

(i) $\mathrm{E}(X)$;

(ii) $\mathrm{Var}(X)$. [5]

(d) Given that $\mu = \mathrm{E}(X)$ and $\sigma^2 = \mathrm{Var}(X)$, find $\mathrm{P}(\mu - \sigma < X < \mu + \sigma)$. [3]

Do **not** write solutions on this page.

12. [Maximum mark: 21]

The equations of two lines, L_1 and L_2, are given by:

$$L_1 : r_1 = \begin{pmatrix} 5 \\ 4 \\ 2 \end{pmatrix} + s\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \text{ where } s \in \mathbb{R}$$

$$L_2 : \frac{x+1}{2} = y - 7 = \frac{z+5}{3}$$

(a) Show that the position vector of the point of intersection, L_1 and L_2 is $\begin{pmatrix} 1 \\ 8 \\ -2 \end{pmatrix}$. [3]

The plane Π contains the lines L_1 and L_2.

(b) (i) Write down the equation of Π, giving your answer in the form $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b} + \mu\boldsymbol{c}$ where $\lambda, \mu \in \mathbb{R}$.

(ii) Given that $\boldsymbol{b} \times \boldsymbol{c} = \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix}$, show that the Cartesian equation of Π is $4x + y - 3z = 18$. [3]

The plane intersects the coordinate axes at $\mathrm{P}(4.5, 0, 0)$, $\mathrm{Q}(0, q, 0)$, and $\mathrm{R}(0, 0, r)$.

(c) Write down the value of

(i) q;

(ii) r. [2]

(d) Use a vector method to find the area of the triangle PQR. [5]

Another line, L_3, is normal to Π and passes through the point of intersection of L_1 and L_2.

(e) Write down an equation for L_3 in the form $\boldsymbol{r}_3 = \boldsymbol{m} + \gamma\boldsymbol{n}$. [2]

(f) Given that the point $\mathrm{S}(-11, 5, 7)$ lies on the line L_3, find γ. [2]

(g) Hence, find the volume of pyramid PQRS. [4]